

USE OF ARTIFICIAL NEURAL NETWORKS TO PREDICT 3-D ELASTIC SETTLEMENT OF FOUNDATIONS ON SOILS WITH AN INCLINED BEDROCK

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Abstract

Application of the theory of elasticity for the calculation of foundation settlements yields equations that are well-established and consolidated in geotechnical standards and/or recommendations. These equations are corrected by an influence factor to increase precision and encompass the existing complex geotechnical casuistry. The study presented herein utilizes neural networks to improve the determination of the influence factor (I_a), which considers the effect of a finite elastic half-space limited by an inclined bedrock under the foundation. The results obtained with the utilization of artificial neural networks demonstrate a notable improvement in the predicted value of the influence factor in comparison with existing analytical equations.

Keywords: artificial neural networks (ANN); foundations; soil/structure interaction; settlement; elasticity; finite-element modelling.

24

25 **1. Introduction**

26

27 The main objectives of a geotechnical engineer, when completing a foundation design, are to
28 determine the safety factor, while guaranteeing adequate functionality and economics. The safety
29 factor is determined by calculating the allowable load. Settlement calculation leads to adequate
30 functionality of the foundation for the actual working pressures, which are usually one-third of
31 the ultimate bearing capacity of the soil. Finally, the economic aspect of the design is very
32 important, obviously always within the safety limits recommended by the standards.

33 Therefore, proper foundation design must ensure that the structure does not suffer excessive
34 displacements. In this sense, the current standards (e.g. CEN 2004) consider the Serviceability
35 Limit State (SLS), which delimits the conditions beyond which the structure no longer fulfils the
36 requirements of functionality, comfort, durability or appearance. Generally, SLS refers to
37 situations that are solvable, repairable or that can admit remedial measures without serious
38 inconveniences to the users. The verification of SLS in a shallow foundation consists of
39 quantifying the total settlement of the foundation and the angular distortion between two adjacent
40 columns and verify whether these values exceed the maximum allowable limits.

41 Therefore, it is very important to evaluate, as accurately as possible, the deformations of the
42 supporting soil for the adopted working pressure levels.

43 Nowadays there are very sophisticated methods for the calculation of shallow foundation
44 settlements, but analytical methods based on the theory of elasticity ("elastic methods") are still
45 widely used in geotechnical practice (mainly during early design phases), and present in all
46 geotechnical standards and recommendations (e.g. CEN 2004; AASHTO 2017). Elastic methods
47 offer versatile solutions that can be easily obtained through laboratory and/or field tests.
48 Therefore, more accurate prediction of settlements depends on the improvement and
49 complementation of existing equations. Although elastic methods are not the best predictors of
50 soil behaviour, when considering working loads far from failure (e.g., shallow foundations where

a safety factor of 3 is accepted), these methods provide a more than acceptable prediction, as demonstrated by the more than 200 real cases studied by Burland *et al.* (1985).

There are three main categories of methods for the computation of the elastic settlement in a shallow foundation:

- Empirical methods, which are based on the compilation and correlation of settlement measured in structures and load tests with the results from in situ data (e.g., SPT, CPT, pressuremeter, etc.). The procedures developed by Terzaghi *et al.* (1967), Meyerhof (1965) and Burland *et al.* (1985) are included in this category.
- Semiempirical methods, which combine field observations and theoretical studies. This category includes, among others, the methods proposed by Schmertmann *et al.* (1978), Briaud (2007) and Akbas and Kulhawy (2009).
- Methods based on theoretical solutions supported by the theory of elasticity, such as those developed by Bowles (1987) and Mayne and Poulos (1999).

All the equations based on the elasticity theory present a similar structure. The general expression for calculating the elastic settlement of a foundation that transmits uniform pressure distribution (q_{net}) resting on an elastic, homogeneous and isotropic medium is (Mayne and Poulos 1999):

$$s = q_{net} B \frac{(1-\nu^2)}{E} I \quad (1)$$

where s is the settlement of the foundation, B is the foundation width, E is Young's modulus for the soil, ν is Poisson's ratio of the soil and I is the displacement influence factor. Displacement influence factors are coefficients that modify the general equation and adapt it to specific cases not covered by the general equation, improving its accuracy. This, when employing elastic methods, the use of I is absolutely necessary to improve the prediction of settlements. Therefore it is very important to develop new displacement influence factors to broaden the application of

elastic methods. A comprehensive explanation on most of the existing displacement influence factors can be found in Milovic (1992) and Mayne and Poulos (1999).

There is limited scientific literature focused on the proposal of influence factors that consider a shallow foundation, with specific stiffness, that rests on an elastic finite half-space with inclined bedrock (i.e. two-layer model, with a deformable layer over a rigid inclined layer). Han *et al.* (2007) carried out the most detailed study of the problem up to date, using the finite difference method. The authors applied a two-dimensional plane strain model to investigate the influence of an inclined incompressible layer (bedrock) on the settlement of a purely flexible strip load on a compressible soil layer. The study highlighted the importance of considering the actual inclination (i.e. the dip) of the rigid layer in settlement calculations. Unacceptable results, from a settlement viewpoint, could be obtained if the dip is not taken into account (resulting in tilting or inadequate differential settlements in the foundation). However, the study applied a load directly to the ground surface (i.e. without considering any element of the foundation with a specific stiffness) and therefore is limited to this specific situation. Nowadays, most foundations present a specific stiffness, being perfectly flexible in very few cases, and therefore it is necessary to study the problem including the consideration of foundation stiffness. Foundation stiffness is evaluated by the foundation flexibility factor (K_f) proposed by Brown (1969). K_f is one of the most widely used parameters to define the stiffness of a shallow foundation and it is defined as follows:

$$K_f = \left(\frac{E_c}{E_s} \right) \left(\frac{t}{a} \right)^3 \quad (2)$$

where E_c refers to the elastic modulus of the foundation material (i.e. concrete), E_s is the representative elastic modulus of the soil beneath foundation base (i.e. value at a depth $z=a$), t is foundation thickness, and a is the equivalent radius of the foundation.

According to the value of the foundation flexibility factor (K_f) the foundations can be considered perfectly rigid when $K_f > 10$, perfectly flexible when $K_f < 0.01$ and intermediately flexible for K_f values between 0.01 and 10.

Díaz and Tomás (2016) analysed the influence of an inclined rigid layer (i.e. bedrock) on the elastic settlements of a shallow foundation. Two-hundred and seventy-three 3D non-linear finite element models were developed considering the foundation stiffness (K_f), inclination (α), and the depth of the rigid layer (z) as variables. Statistical analysis of the results enabled the proposition of an analytical equation that can be applied to the calculation of settlements.

Artificial Neural Networks (ANN) employ artificial intelligence to simulate the biological structure of the human brain and nervous system through their architecture. This concept was firstly introduced in 1943 by McCulloch and Pitts (1943), and expanded by Werbos (1974) through the development of the backpropagation algorithm, becoming a practical tool in the field of forecasting and prediction.

ANN have been successfully applied to several geotechnical engineering problems during the last decades (e.g. Zounemat-Kermani *et al.* 2009; Tarawneh 2013; Mozumder and Laskar 2015; Ochmański *et al.* 2015; Benali *et al.* 2017). More specifically, ANN have also been used for the prediction of foundation settlements, with highly satisfactory results (e.g. Shahin *et al.* 2002; Zhang *et al.* 2011; Shahin 2014; Shahin 2014; Baziar *et al.* 2015; Harikumar *et al.* 2016).

The objective of the study presented herein is to apply ANN to predict 3-D elastic settlements of shallow foundations on soils with a rigid inclined layer. From a database containing 273 registries derived from Finite Element Method (FEM) models, 212 (77.4%) were used to train the neural network and the remaining 61 (22.6%) were used to test the network. ANN predictions were then compared with the predictions obtained from the application of the equation recently proposed by Díaz and Tomás (2016), based on traditional analytical data-fitting derived from the FEM 3D models.

2. Methodology

2.1. FEM model

FEM software ANSYS V.11 (Ansys 2007a; Ansys 2007b; Ansys 2007c) was used to model the case in which a shallow foundation rests on an elastic finite half-space (i.e., a compressible layer over a rigid layer). A 3D nonlinear model with contact elements between the foundation and the soil was developed to simulate the foundation-soil interface through the Mohr-Coulomb law. These contact elements enable the consideration of the friction between materials. Herein an interface friction angle equal to 2/3 of the friction angle of the soil was considered. This value, recommended by Potyondy (1961), is commonly accepted for the computation of soil-concrete friction.

A refinement of the mesh in the zone below the foundation (higher concentration of stress) was implemented, and elements with a size of 1/10 of the foundation width were used in this zone. The size of the elements was progressively increased to 1/2 of the foundation width in the limits of the model.

The model presents a deformable top layer over rigid soil that can be considered incompressible (i.e. rigid) for the normal range of pressures present in shallow foundations. This layer, located at depth z , presents a specific inclination (α) with respect to the horizontal. Figure 1 depicts a scheme of the model adopted.

The finite element models were solved by varying the key variables of the problem. Table 1 shows the specific parameters for which the models were solved for. Combination of these values yielded 273 finite element models.

Variable	Values								
z/B	0.25	0.5	0.75	1	1.5	2	3	5	10
α (°)	0	15	30	45	60				
K_f	0.001	0.01	0.1	1	10	30	100		

Table 1. Values of the variables used in the FE models.

Further details on the model and procedure carried out can be found in Díaz and Tomás (2016).

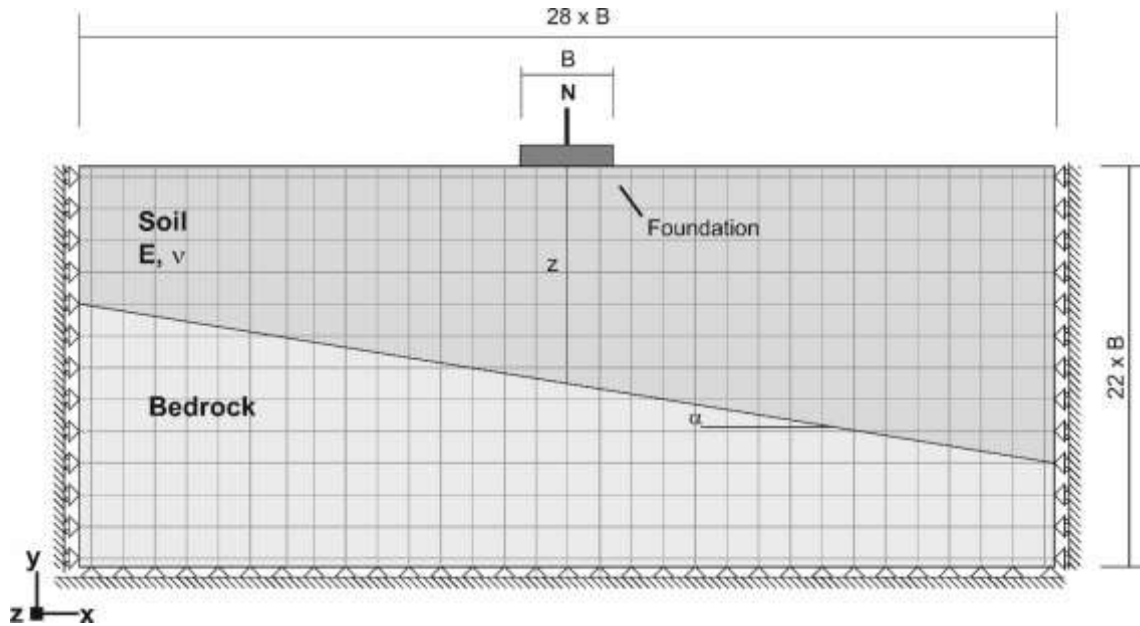


Fig. 1. Geometry of the 3D FEM model adopted for modelling the settlement of a foundation resting on an elastic finite half-space with an inclined bedrock (rigid layer) at specific depth.

2.2. Neural networks

ANN systems mimic the behaviour of the human brain, in the sense that they require a learning process, accomplished from input datasets and known outputs, to predict results (output data) associated with cases (input data) not used in the training. Utilization of a higher number of cases for training enables more reliable predictions of the network. Another analogy with the brain can be made by considering the internal structure or network architecture. There are many types of network architectures, but basically all are constituted of nodes or perceptrons (neurons in the brain) with connections (synapses) between neurons. Each neuron processes the input signals received from other neurons and transmits the result to their output neurons. The concept of neural networks is very old, but its massive application is much more recent.

One of the network architectures most commonly used is the feedforward neural network. Figure 2 shows a scheme of the structure of a very simple feedforward neural network, consisting of three layers of neurons.

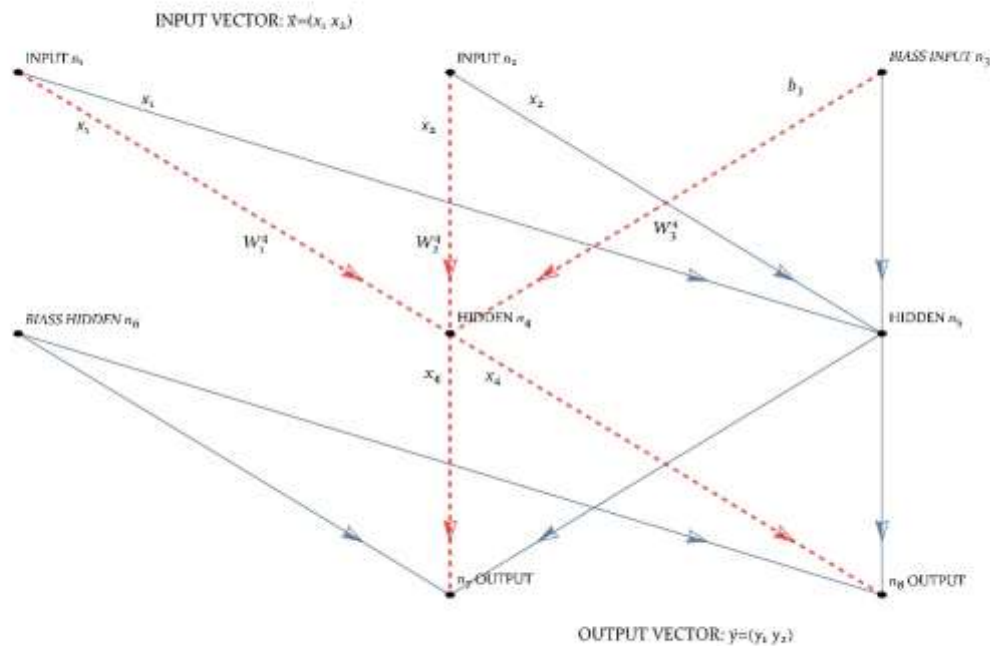


Fig. 2. A feedforward neural network with one hidden layer. n_1, n_2 : input-neurons; n_3, n_6 : Bias-neurons; n_4, n_5 : hidden layer neurons; n_7, n_8 : output-neurons.

Two of these layers, input and output, are always present, with any number of hidden layers (one in this case) in between. The connection structure is as follows: any neuron is fully connected to all neurons in the previous layer for receiving data, and with all the neurons in the next layer for transmitting the result processed. The exceptions are BIAS neurons that do not receive input data and are used to correct the bias. Neural network learning is accomplished by automatic fitting, through an iterative process of the synaptic weights. While neurons transmit resulting processed data to all outbound connections, these data are weighted independently at each connection by the corresponding synaptic weight, so that each neuron connected to the output receives a different

value. The processing of the sum of input data by each neuron is performed by a nonlinear activation function, which can take several forms and response settings within each concrete form. The advantage of using neural networks rather than analytical function fits (such as least-squares methods), becomes clear when considering that in this case there are 12 synaptic weights or fitting parameters. These fit parameters can be easily expanded by increasing the number of neurons and hidden layers, and are covered by the nonlinear input-output dependencies through the activation function. During the learning process, synaptic weights are automatically adjusted through an iterative process that seeks the minimum MSE between the target data (actual) and the network output data for the same inputs. In compact form, the response of an active neuron (not BIAS) can be written as Equation (3).

$$x_j = \sigma \left(\sum_{i=m}^{n-1} x_i w_i^j + b_n^j \right) \quad (3)$$

where x_j is the result of neuron j of layer k, $\sigma(x)$ is the activation function, m is the number of the first neuron in the previous layer, n is the number of the first neuron in the previous layer (BIAS), x_i is the result of neuron i of layer k-1, w_i^j is the synaptic weight of i, j connection, and b_n^j is the BIAS weight connection.

It is worth noting that ANN have been considered as an alternative to traditional analytical fitting in detriment of other techniques of supervised machine learning (e.g. support vector machines), because ANNs achieved prediction errors that were considered adequate for the purposes of the present work.

3. Prediction of 3-D elastic settlements by means of neural networks

A total of 273 non-linear finite element models were solved to analyze the relationship (I_a) between the settlement of a shallow foundation on an infinite elastic half-space (s_∞) and the

highest settlement obtained for the same foundation resting on a deformable layer over an inclined rigid layer (s_a). The results were used to implement an ANN to obtain an approximation function that enables the prediction of settlements.

The network used herein consists of an input layer with three neurons (plus one BIAS neuron), hidden layer 1 with 36 neurons, hidden layer 2 with 18 neurons and hidden layer 3 with nine neurons. Each hidden layer has one additional BIAS neuron to correct the bias. The output layer contains three neurons. The total number of neurons is 73 and, with the used network topology, 1011 synaptic connections were created (Figure 3).

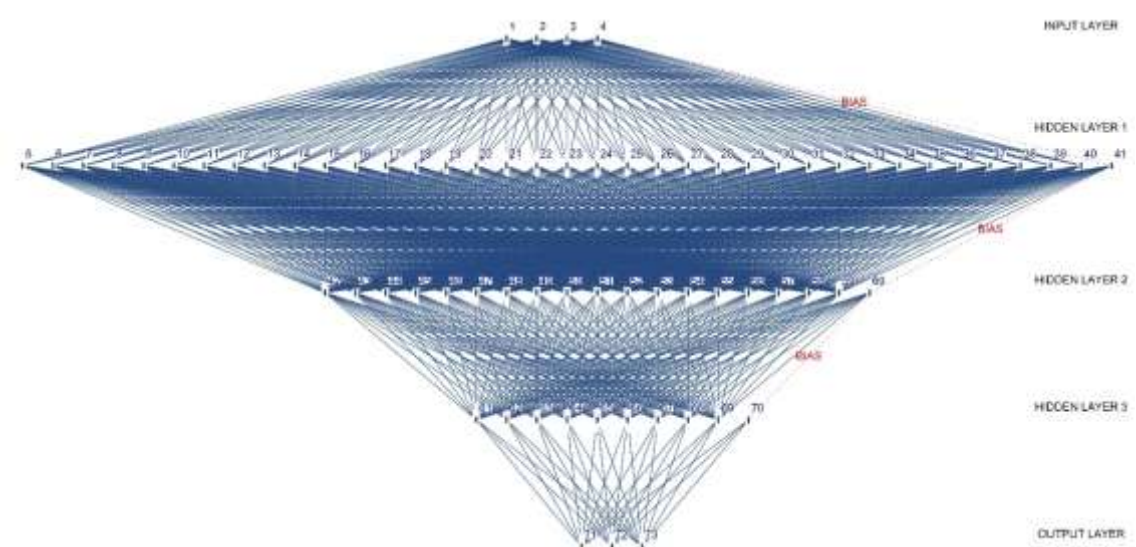


Fig. 3. Overview of the neural network used.

In Figure 3, the numbers in the boxes indicate the number of the neuron. Of the five layers of neurons, the topmost and the bottommost constitute the input and output layers, respectively. The three intermediate layers correspond to hidden layers.

Although there are studies about the determination of the optimal size and architecture of the network (e.g. Hunter *et al.* 2012), the process carried out in herein focused on fixing the number of layers and neurons, through a previous study where different network configurations were analysed. In this analysis, the values of MSE (mean squared error), MRE (Mean Relative

Estimation Error between the predicted and the target data) and R^2 (coefficient of determination) obtained for each configuration were compared, followed by the selection of the configuration with better values. In addition, the network designed herein can be used with a higher volume of data in the future.

The high number of connections and neurons generated a neural network with high fitting capability (1011 synaptic weights). Input vectors present three components, coinciding with the number of neurons in the data input layer (without the BIAS neuron), representing the input variables of each case (K_f , α , z/B) previously defined. The output vectors also present three components, for topology requirements of this type of neural network, although the desired result (I_α) is unidimensional. In each case, I_α is obtained as the mean value of the three components of the output vector.

On the other hand, in this work, the sigmoid symmetric activation function $\sigma(x)$ has been used in the interval $[-1, +1]$, as is shown in Equation (4).

$$\sigma(x) = \frac{1}{1+e^{-sx}} \quad (4)$$

where s is the parameter that adjusts the smoothness of the response function.

Although the activation function selected was the sigmoid symmetric, several activation functions were previously tested. The function with the lowest MSE value and successful convergence was selected.

4. Analysis of results

A dataset with 273 input-output vectors corresponding to the relationship between the settlement of a shallow foundation resting on an infinite elastic half-space and the highest settlement

obtained for the same foundation on a deformable layer over an inclined rigid layer was used in the implementation of ANN. ANN was trained with 212 randomly selected data vectors, followed by an accuracy test for the trained network, carried out with the remaining 61 vectors. These two steps correspond to the two mandatory phases required before using a neural network: training and validation.

Regarding the input database used to train the network, a brief characterization with its main statistics is presented in Figure 4.

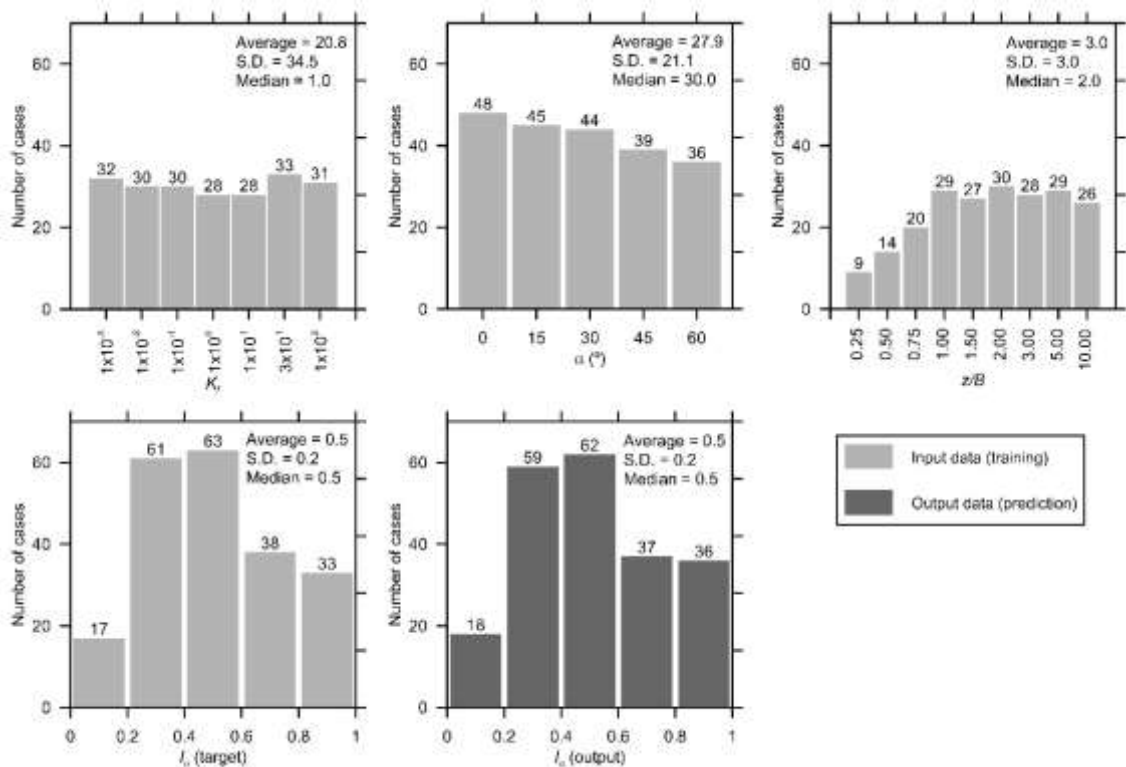


Fig. 4. Histograms of the input and output variables. S.D. refers to standard deviation.

It is worth noting that, to avoid overfitting, the neural network was trained with 20 sets of randomly chosen input data. For each series, five re-trainings were carried out. The synaptic weights selected for this process were those that presented the lowest MSE values. However, it should be noted that the differences obtained were negligible and after $4 \cdot 10^5$ iterations in all re-trainings, MSE was always under $1.5 \cdot 10^{-4}$.

Figure 5 shows the results of the network training, providing MSE and MRE values equal to $1.26 \cdot 10^{-4}$ and 2.04%, respectively. Figure 5 represents the linear regression between the target data (used in the training) and the output data network, obtained from the trained network using the corresponding input parameters.

The fitting equation provided an R^2 value of 0.997.

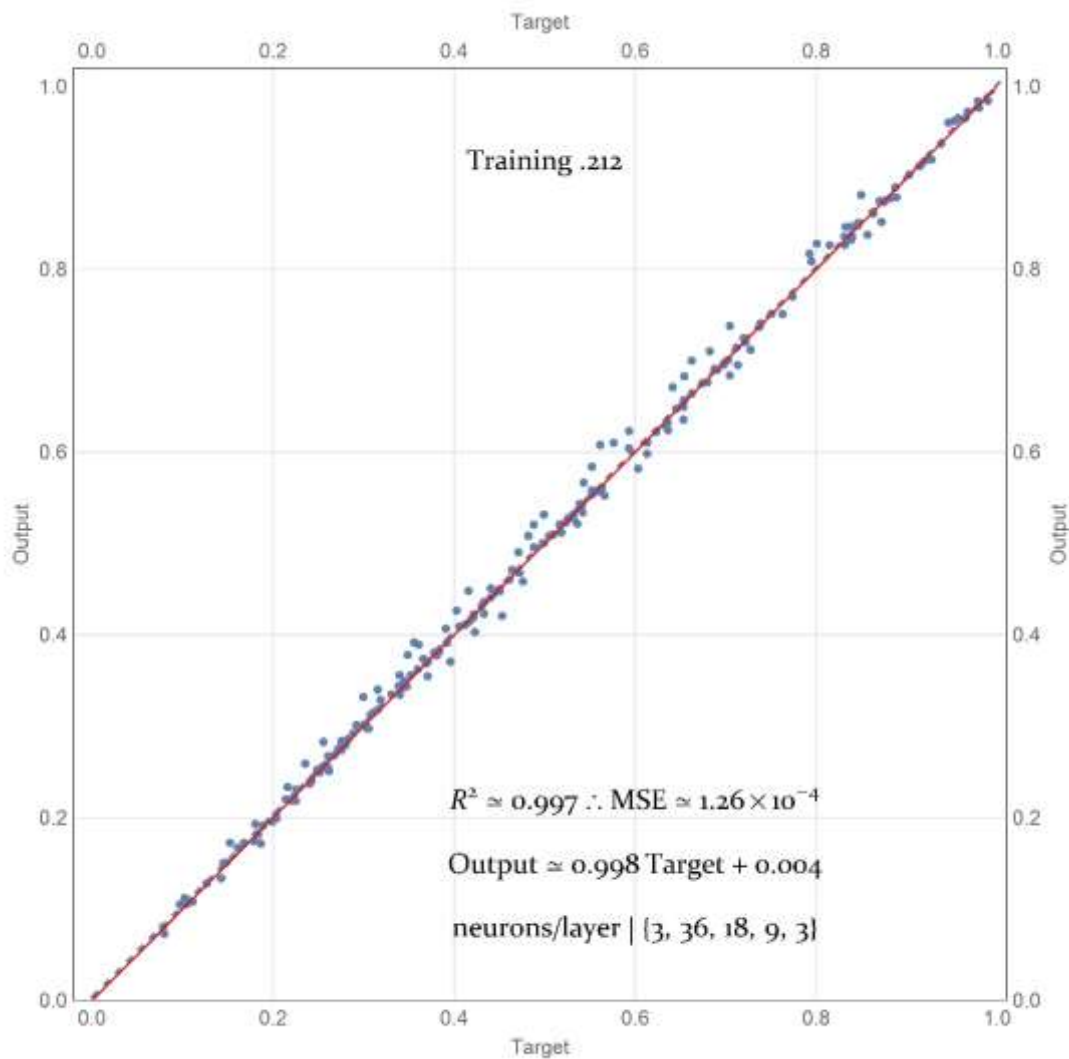


Fig. 5. Comparison and regression of actual settlements (Target) and artificial neural network predicted settlements (Output) using training data for 212 study cases.

Figure 5 also shows that the points line up very well with a straight line that crosses the origin with a slope near 1 (i.e. 45 degrees). This indicates high quality parameter fitting of the neural network during the training.

Once the artificial neural network was trained, the next step consisted of the application of a test to the 61 vectors not used for training, with the results shown in Figure 6.

The values of the foundation settlement predicted by ANN provided MSE value equal to $1.48 \cdot 10^{-4}$, R^2 equal to 0.998 and a fitting line slope of 1.000 (i.e. 45°).

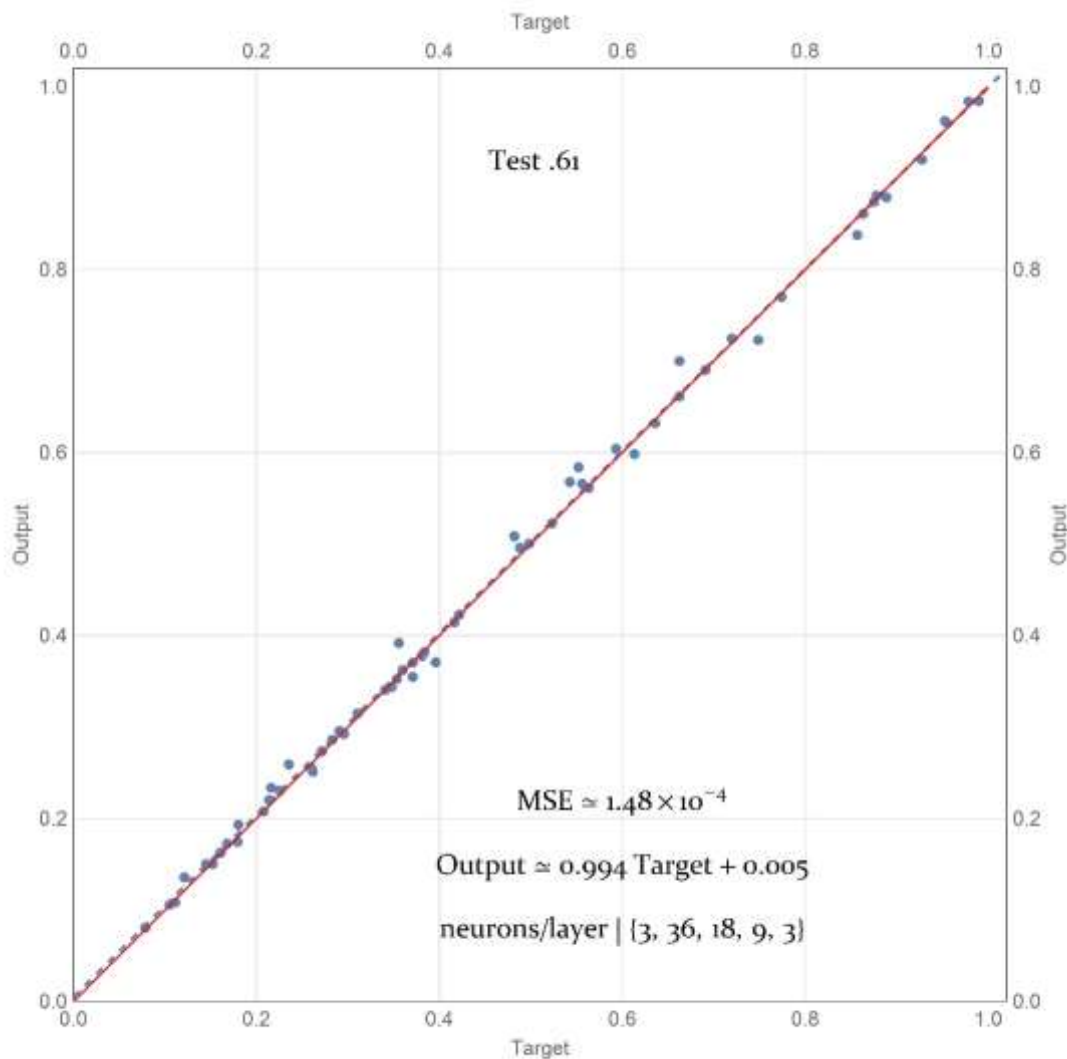


Fig. 6. Comparison of actual settlements (Target) and the settlements predicted by means of the artificial neural network (Output) for 61 study cases (Test).

An application was created with the trained network to obtain the output values (I_α) from any set of input values (K_f , α , z/B). K_f is the foundation flexibility factor, according to Brown (1969), α and z are the dip and the depth of the rigid layer under the central point of the foundation, respectively, and B is the foundation width, as previously described. Figure 7 shows the appearance of the application generated from the specific training used herein.

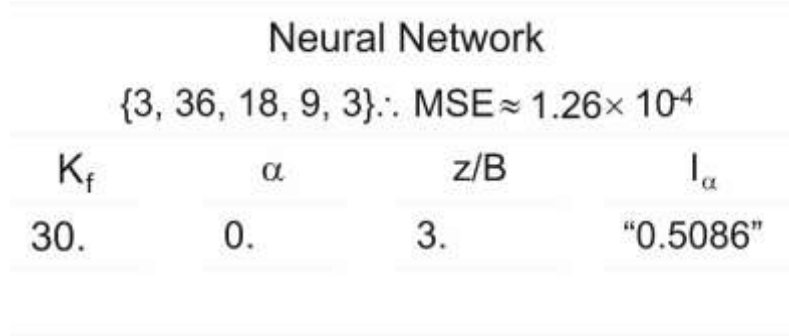


Fig. 7. Application with the trained network.

The equation obtained in the original analytical fitting (Díaz and Tomás 2016) was:

$$I_\alpha = 0.1261 \cdot e^{-(8.7510 \cdot K_f + 0.0949)} + \left(\frac{1}{1529} \cdot \alpha + 0.7690 \right)^2 - 1.1715 \cdot e^{-(0.4892 \cdot \frac{z}{B} + 0.7061)} - 0.0002 \cdot K_f + \frac{z}{B} \cdot 0.0249 \quad (5)$$

Using data from the trained network, the target-prediction regressions of both fits (analytical and ANN) were computed, followed by a comparison of accuracies. Figure 8 shows the point clouds, and the MSE and MRE values. In both cases, the errors obtained with the trained ANN were lower than the errors obtained with the analytical fit.

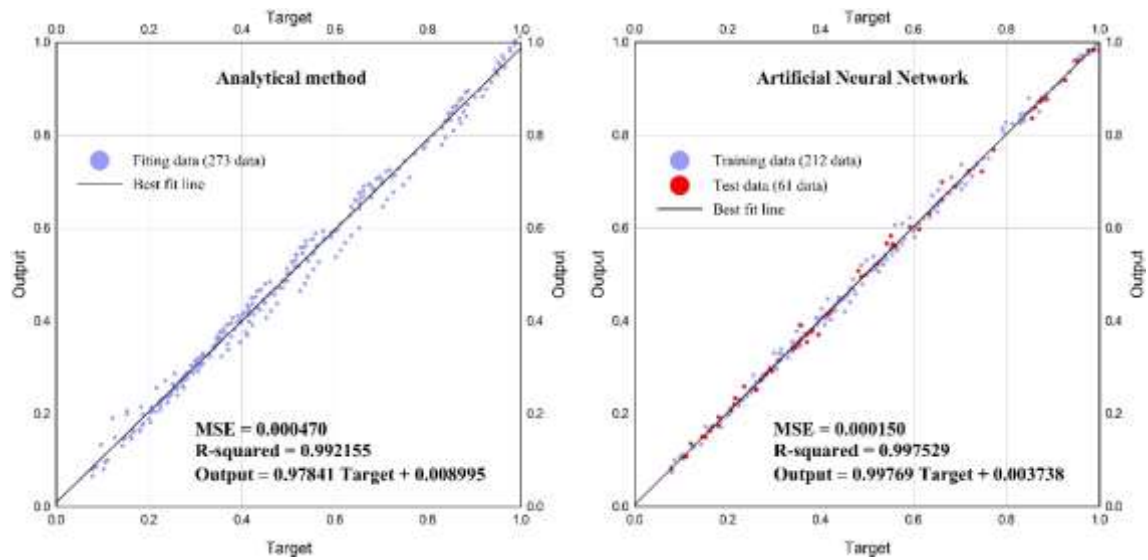


Fig. 8. Comparison of the fitting performed over the results provided by: (left) the analytical method proposed by Díaz and Tomás (2016); and (right) trained ANN.

Table 2 summarizes the main results of the fitting. Prediction of foundation settlement by means of ANN provided better results for all values and evaluated statistical parameters.

Parameter	Analytical fit	ANN
Coefficient of determination (R^2)	0.992	0.997
Mean quadratic error (MSE)	$4.70 \cdot 10^{-4}$	$1.26 \cdot 10^{-4}$
Mean relative error (MRE)	4.19%	2.04%

Table 2. Comparison of the statistical parameters obtained for the analytical fit and the ANN (training).

5. Design charts

The results obtained from the trained ANN were conveniently organized and represented in six design charts to quickly estimate I_α for different values of the input variables (K_f , α , z/B). Figure 9 shows these design charts.

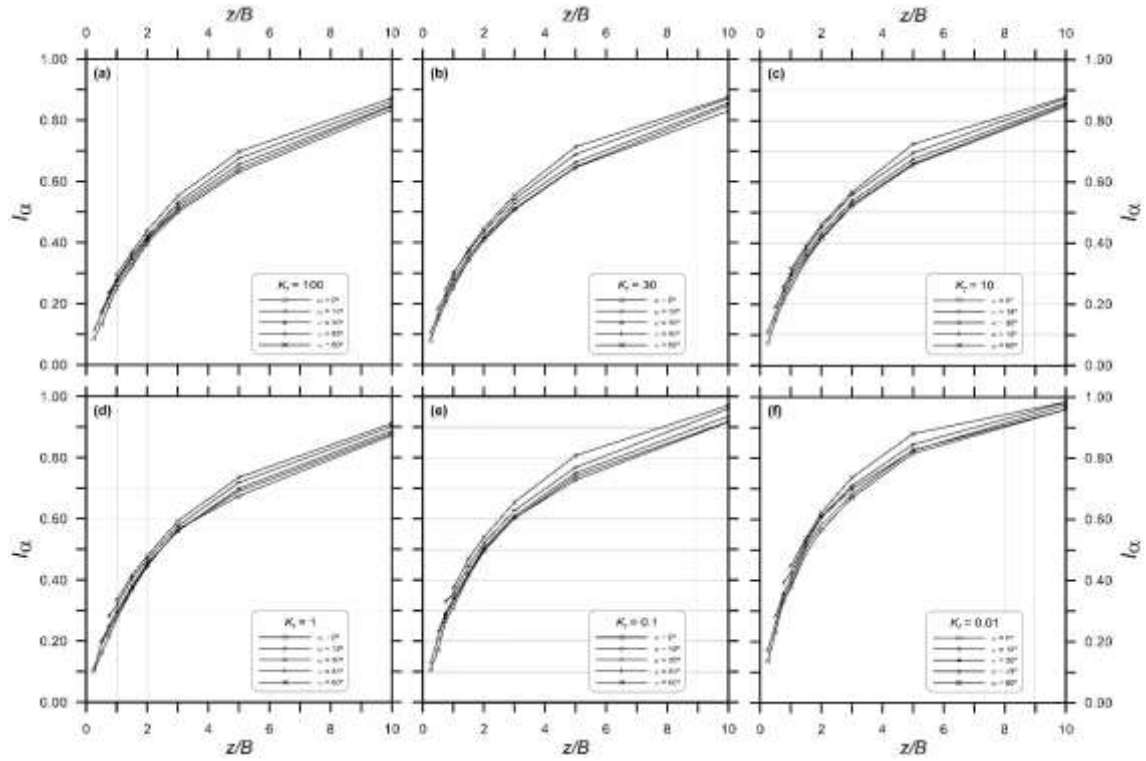


Fig. 9. Design charts for the calculation of I_α from different z/B ratios and α values. The figure covers the following K_f values: a) 100, b) 30, c) 10, d) 1, e) 0.1 and f) 0.01.

6. Conclusions

The study presented herein predicted the settlement of foundations resting on a finite half-space with an inclined rigid layer, by means of artificial neural networks. This study demonstrated the feasibility of ANN to predict the settlement of shallow foundations under these conditions. The ANN was developed with 273 results of 3D non-linear FEM models, of which 212 corresponded to training and 61 to testing. Subsequently, an application was developed with the trained network to obtain I_α from any set of input values (K_f , α , z/B). These I_α predicted values were compared with those obtained from the analytical fitting of the results of FEM models. It was verified that ANN was capable of accurately predicting the settlement of foundations resting on a finite half-space with an inclined rigid layer.

The results also established that the ANN method provides better results than traditional analytical regression methods (squared coefficient of determination equal to 0.997, mean quadratic error equal to $1.26 \cdot 10^{-4}$ and mean relative error equal to 2.04%).

Furthermore, six synthetic design charts were built using the input and output parameters derived from the ANN. These charts relate I_a with other key parameters (K_f , α , z/B), enabling I_a estimations that can be utilized at design stages.

Finally, it must be highlighted that artificial neural networks present another advantage over traditional regression methods: once the model has been trained and tested, it can be utilized as an accurate and quick tool for the estimation of settlement under the conditions studied herein.

Acknowledgments

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